Modelling of thunderstorm outflows by means of the evolutionary power spectral density

Luca Roncallo [a], Giovanni Solari [a]

[a]*Department of Civil, Chemical and Environmental engineering (DICCA)*
Polytechnic School, University of Genova, Via Montallegro, 1, 16145 Genova (Italy)

ABSTRACT: This study proposes a model of time-modulating function for the evolutionary power spectral density (EPSD) of thunderstorm outflows, based on the slowly-varying wind speed and standard deviation records extracted from a database of 137 10-minutes thunderstorm time-histories. The analyses have been carried out under the hypothesis that the residual turbulent fluctuations can be considered an uniformly modulated process. Three different models have been proposed and provided with parameters characteristic of both the thunderstorm event and the background wind. Said models have been adopted to fit the ensemble value and its variation with the standard deviation of the dimensionless functions of both the slow-varying mean velocity and standard deviation records. Successively the relations between parameters have been provided and the procedure for the derivation of the EPSD have been outlined. The aim is to adopt this model for the derivation of the dynamic response of structures to thunderstorms outflows.

KEYWORDS: Thunderstorm, Evolutionary power spectral density, time-modulating function.

1 INTRODUCTION

The evolutionary power spectral density is a valuable tool for depicting the non-stationary nature of the residual turbulent fluctuations of thunderstorm outflows, however it suffers the drawback of being turned into rapid engineering calculations. This study aims to derive a new and simplified approach for the evaluation of the EPSD of thunderstorm outflows on which the derivation of the dynamic response can be based on. Starting from the hypothesis of uniformly modulated process [1], the focus of the study is to model the slowly-varying mean wind velocity and standard deviation of the thunderstorm outflows to be employed as time-modulating functions of the EPSD. Previous work carried out by Chen [2] proposed different modulation functions for the time varying mean wind speed which include a numerically simulated function based on an empirical model and two functions obtained from full-scale data. Kwon & Kareem [3] proposed an half sine wave to describe the time varying mean component of gust-front winds with the aim of capturing the features potentially responsible for enhanced loads on structures. While these attempts are based on few records or on physical assumptions, the model proposed in this work is built on a robust database of thunderstorm events, involving parameters characteristic of the outflows and of the background wind that can be characterized from a stochastic point of view.

2 THUNDERSTORM WIND VELOCITY

The classical decomposition of the thunderstorm adirectional wind velocity v reads:

$$v(t) = \bar{v}(t) + v'(t) \tag{1}$$

where $\bar{v}$ is the slowly-varying mean wind speed and v' the residual turbulent fluctuations with zero mean. This latter quantity is usually decomposed as follows:

$$v'(t) = \sigma_v(t)\tilde{v}'(t) \tag{2}$$

where σ_v is the slowly-varying standard deviation of v' and $\tilde{v}'$ is the reduced turbulent fluctuation, dealt with as a stationary Gaussian random process. It follows that, under the hypothesis of uniformly modulated process, the EPSD of the quantity v' can be written as:

$$\hat{S}_{v'}(t,n) = \sigma_v{}^2(t)S_{\tilde{v}'}(n) \tag{3}$$

where $\hat{S}_{v'}$ is the EPSD of the residual turbulent fluctuations and $S_{\tilde{v}'}$ is the PSD of the reduced turbulent fluctuations. According to Equation 3 the slowly-varying standard deviation seems to be the best choice for the role of time-modulating function of the EPSD. However, the possibility of adopting the slowly-varying mean wind velocity has been considered as well, being more suitable and explicit from a physical point of view. Therefore the following dimensionless quantities have been defined [4]:

$$\gamma(t) = \bar{v}(t) / \bar{v}_{\max} \tag{4}$$

$$\delta(t) = \sigma_v(t) / \sigma_{v,\max} \tag{5}$$

where $\bar{v}_{max}$ and $\sigma_{v,max}$ are respectively the maximum values of the slowly-varying mean wind speed and of the slowly-varying standard deviation.

3 THUNDERSTORM DATA

In order to study the behavior of the quantities in Equations 4-5, the time-histories of 137 thunderstorms with duration of 10 minutes recorded in four different Italian ports of Genova, La Spezia, Savona and Livorno (previously analyzed by Zhang [5]) have been considered and their slow-varying mean and standard deviation have been extracted by means of a moving average technique over a time interval of 30s. The functions defined in Equations 4-5 have been evaluated for each event and reported in a way that their maximum value occurs at t=0s. Hence at each point in time a distribution of values for both the quantities γ and δ has been derived, along with its mean value and standard deviation. The behavior of said functions has been studied by adding, at each instant t, a certain amount j of standard deviation to the mean. The results of this operation are reported in Figures 1-2. It is shown that the curves of both the functions result regular and symmetric, however while the ones of γ results particularly smooth, the ones related to δ present a sharp point in a neighborhood of t=±30s. This remarkable aspect points out that from a statistical point of view, although both the functions have similar shapes, they are not the same. Therefore, also in view of Equation 3, it is not reasonable in principle to adopt the slowly-varying mean wind speed as time-modulating function of the EPSD. Further considerations including the peaks related to δ in a neighborhood of t=0s will be reported in the final paper.

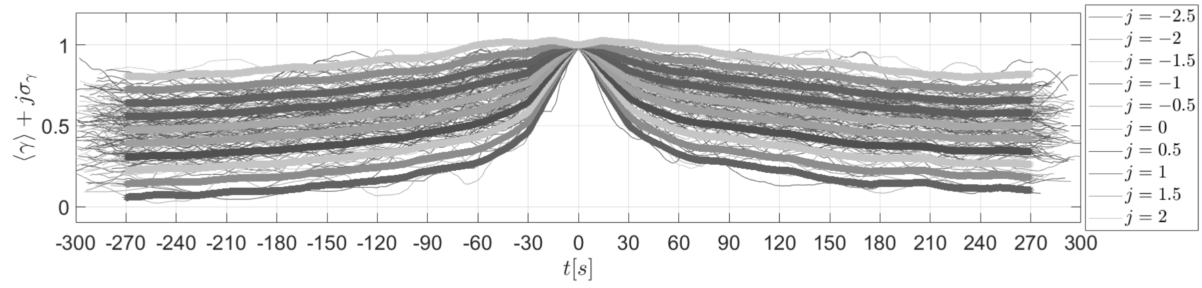

Figure 1. Dimensionless function γ and its variation $\langle\gamma\rangle + j\sigma_\gamma$ over 137 thunderstorms recorded.

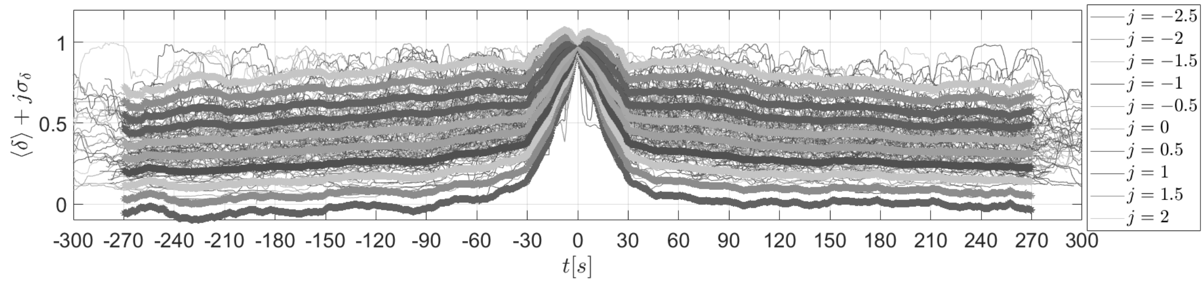

Figure 2. Dimensionless function δ and its variation $\langle\delta\rangle + j\sigma_\delta$ over 137 thunderstorms recorded.

Nevertheless, with the aim of deriving valuable relations between γ and δ, the slowly-varying mean wind speed has been modelled along with the slowly-varying standard deviation.

4 MATHEMATICAL MODEL

Based on the above considerations, three different models have been proposed and read:

$$\phi(t) = \frac{1-\phi^*}{1+(t/T_\phi)^2} + \phi^* \tag{6}$$

$$\phi(t) = \frac{1-\phi^*}{\sqrt{1+(t/T_\phi)^2}} + \phi^* \tag{7}$$

$$\phi(t) = (1-\phi^*)e^{-(t/T_\phi)^2} + \phi^* \tag{8}$$

where $\phi = \gamma,\delta$ and T_ϕ is a measure of the duration of the peak. The parameters γ^* and δ^* have been defined as follows:

$$\gamma^* = \bar{v}^* / \bar{v}_{\max} \tag{9}$$

$$\delta^* = \sigma_v^* / \sigma_{v,\max} \tag{10}$$

where $\bar{v}^*$ and σ_v^* are respectively the mean wind velocity and standard deviation of the background wind. The models are designed so that for t=0s the mean wind velocity and standard deviation reach their maximum while for large values of $|t|$, i.e. wind conditions not influenced by the thunderstorm, they result the ones of the background wind. Successively the models have been adopted to fit the curves in Figures 1-2, providing the parameters at each j position. The models successfully fit the curves, especially the ones in Equations 6-7. The variation of the parameters with j has been investigated as well as the relations between the ones of γ and δ.

5 DERIVATION OF THE EPSD

The EPSD of the residual fluctuations is aimed to be derived adopting the slowly-varying standard deviation as time-modulating function, shaped through one of the models in Equations 6-8 and starting from the characteristic parameters of γ, which are more suitable for being characterized from a stochastic point of view. Therefore the following relations have been provided:

$$T_\gamma = \left(\frac{p}{\gamma^* - 1}\right)^2 + q \tag{11}$$

$$\delta^* = g\gamma^* + k \tag{12}$$

$$T_\delta = ae^{bT_\gamma} + ce^{dT_\gamma} \tag{13}$$

where *a, b, c, d, g, k, p* and *q* depend on which model in Equations 6-8 is chosen and will be outlined in the final paper. Finally, the procedure for the derivation of the EPSD is reported:

1. Define $\bar{v}^*$, $\bar{v}_{max}$ and $\sigma_{v,max}$;
2. Evaluate the parameters γ^*, T_γ and δ^*, T_δ coherently with the model chosen;
3. Derive $\sigma_v(t)$ through Equation 5;
4. Model $S_{\tilde{v}'}$ with spectral models proposed in literature;
5. Reconstruct the EPSD by means of Equation 3.

It is worth to notice that the choice of a spectral model for $S_{\tilde{v}'}$ is under discussion due to the delicate choice of its parameters as it has been pointed out by Zhang [5].

6 CONCLUSIONS AND PROSPECTS

The model presented in this study allows to reconstruct the EPSD of the residual fluctuations of the thunderstorm outflows adopting the slowly-varying standard deviation as time-modulating function, which can be derived starting from the values of the background mean wind velocity, the maximum value of the slowly-varying mean wind speed and the maximum value of the slowly-varying standard deviation. The philosophy behind the three models proposed is to provide a deterministic shape for both the slowly-varying mean and standard deviation while their parameters are aimed to be characterized from a stochastic point of view. The procedure has been developed under the hypothesis of uniformly modulated process, which is aimed to be verified in future studies.

7 ACKNOWLEDGEMENTS

This research is funded by the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation program (grant agreement No. 741273) for the project THUNDERR - Detection, simulation, modelling and loading of thunderstorm outflows to design wind-safer and cost-efficient structures – supported by an Advanced Grant 2016.

8 REFERENCES

1 M.B. Priestley, Evolutionary spectra and non-stationary processes, J. R. Statist. Soc. B, 27 (1965) 204-237.
2 X. Chen, Analysis of alongwind tall building response to transient nonstationary winds, J. Struct. Eng. ASCE, (2008) 782-791.
3 D. Kwon and A. Kareem, Gust-Front factor: New framework for wind load effects on structures, J. Struct. Eng. ASCE, (2009) 717-732.
4 G. Solari *et al.*, Characteristics of thunderstorms relevant to the wind loading of structures, J. Wind and Struct. Vol. 20, 6 (2015) 763-791.
5 S. Zhang *et al.*, A refined analysis of thunderstorm outflow characteristics relevant to the wind loading of structures, Probabilistics Eng. Mech., 54 (2018a) 9-24.